[12-06-04-T8]

Quadratic Formula

QUADRATIC FORMULA.

The solutions of the equation $\boldsymbol{a\ x^2 + b\,x + c = 0,\ a \neq 0}$ are given by the quadratic formula which is

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}}.$$

DERIVATION OF QUADRATIC FORMULA. Suppose $a\,x^2 + b\,x + c = 0,\ a \neq 0$. We solve by completing the square in general.

$a\,x^2 + b\,x + c = 0$ ⋮

$$\Longleftrightarrow x^2 + \frac{b}{a}\,x + \frac{c}{a} = 0$$

$$\Longleftrightarrow x^2 + \frac{b}{a}\,x = -\frac{c}{a}$$

$$\Longleftrightarrow x^2 + \frac{b}{a}\,x + \frac{b^2}{4\,a^2} = \frac{b^2}{4\,a^2} - \frac{c}{a}$$

$$\Longleftrightarrow \left(x + \frac{b}{2\,a}\right)^2 = \frac{b^2}{4\,a^2} - \frac{c}{a}$$

$$\Longleftrightarrow x + \frac{b}{2\,a} = \pm\sqrt{\frac{b^2}{4\,a^2} - \frac{c}{a}}$$

$$\Longleftrightarrow x = -\frac{b}{2\,a} \pm \sqrt{\frac{b^2}{4\,a^2} - \frac{c}{a}}$$

$$\Longleftrightarrow x = -\frac{b}{2\,a} \pm \sqrt{\frac{b^2}{4\,a^2} - \frac{4\,a\,c}{4\,a^2}}$$

$$\Longleftrightarrow x = -\frac{b}{2\,a} \pm \frac{1}{2\,a}\sqrt{b^2 - 4\,a\,c}$$

$$\Longleftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

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